WORKING PAPER

A Hypothetical General Law for the Coefficients Cn

Perturbative Series for the Proton-to-Electron Mass Ratio

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Status: Working hypothesis. Mathematically consistent with known coefficients (n = 0 to 3). Not experimentally verifiable with CODATA 2022. Potentially testable with CODATA 2026-2028. Not submitted for publication.

1. Background

In a previously published preprint [1], we reported the following perturbative series for m_p/m_e:

$$\mathbf{mp/me = C_0 + C_1\alpha + C_2\alpha^2 + C_3\alpha^3 + ...}$$

with the following known coefficients, verified within CODATA 2022 uncertainty:

$C_0 = 6\pi^5$ factor $a_0 = 6$, π-exponent $b_0 = 5$

$C_1 = (3/2)\pi$ factor $a_1 = 3/2$, π-exponent $b_1 = 1$

$C_2 = \pi$ factor $a_2 = 1$, π-exponent $b_2 = 1$

$C_3 = \pi^4/4$ factor $a_3 = 1/4$, π-exponent $b_3 = 4$

2. Observed Recurrence

Examining the successive ratios between factors:

$a_1/a_0 = 1/4$

$a_2/a_1 = 2/3$

$a_3/a_2 = 1/4$

The ratios alternate: 1/4, 2/3, 1/4, 2/3... Since (1/4) × (2/3) = 1/6, the complete cycle gives:

$$\mathbf{f_{n+2} = (1/6) \times f_n}$$

3. Closed-Form Solution

The recurrence $f_{n+2} = (1/6)f_n$ is a second-order linear recurrence. Its general solution is:

$$\mathbf{f_n = A \times (1/\sqrt{6})^n + B \times (-1/\sqrt{6})^n}$$

Solving from the initial conditions $f_0 = 6$ and $f_1 = 3/2$:

$$A + B = 6$$

$$(A - B)/\sqrt{6} = 3/2 \Rightarrow A - B = 3\sqrt{6}/2$$

$$\mathbf{A = (6 + 3\sqrt{6}/2)/2 = 3(1 + \sqrt{6}/4) \approx 4.837}$$

$$\mathbf{B = (6 - 3\sqrt{6}/2)/2 = 3(1 - \sqrt{6}/4) \approx 1.163}$$

Verification

$f(0) = A + B = 6$ ✓

$f(1) = (A-B)/\sqrt{6} = 3/2$ ✓

$f(2) = (A+B)/6 = 1$ ✓

$f(3) = (A-B)/(6\sqrt{6}) = 1/4$ ✓

$f(4) = (A+B)/36 = 1/6$ ✓ (predicted $C_4 = \pi^7/6$)

Remark on √6

$\sqrt{6} = \sqrt{(2\times3)} = \sqrt{(Nf \times Nc)}$, where Nf = 2 (light quark flavors u, d) and Nc = 3 (QCD colors). This is the same factor 6 = Nf × Nc that appears in the dominant term $C_0 = 6\pi^5$. Whether this is a meaningful connection or a numerical coincidence remains an open question.

4. Law of π-Exponents b_n

For $n \geq 2$, successive differences of b_n are +3, +3... giving:

$$\mathbf{b_n = 1 + 3(n-2) \quad for\ n \geq 2}$$

with $b_0 = 5$ (special dominant term) and $b_1 = 1$.

5. Complete Hypothetical Formula

Combining the closed-form for f_n and the law for b_n, the complete series reads:

$$\mathbf{C_n = [A \times (1/\sqrt{6})^n + B \times (-1/\sqrt{6})^n] \times \pi\text{^}b_n}$$

with A, B as above and $b_n = 1 + 3(n-2)$ for $n \geq 2$.

Predictions

$C_4 = \pi^7/6$ → testable with CODATA 2026

$C_5 = \pi^{10}/10$

$C_6 = \pi^{13}/36$

6. Honest Assessment

The sum of the series up to n=7 using this law yields 1836.15267495, a deviation of 1.52×10^{-6} from CODATA 2022 — slightly larger than the series truncated at order 3 alone (deviation 4×10^{-12}). This suggests that while the recurrence captures the structure of the known coefficients, the predicted higher-order terms may not be exactly correct.

The formula is mathematically complete and self-consistent. Its physical validity beyond n=3 awaits experimental confirmation.

7. Next Steps

- Monitor CODATA 2026 — to test $C_4 = \pi^7/6$
- Contact PENTATRAP group for high-precision mp/me measurements
- Seek physical origin of the recurrence $f_{n+2} = (1/6)f_n$
- If C_4 confirmed → derive unified law for both b_n and f_n

References

[1] J. Brindel, «Perturbative Series for the Proton-to-Electron Mass Ratio», Zenodo (2026). DOI: 10.5281/zenodo.18693489

[2] CODATA 2022, NIST. https://physics.nist.gov/cuu/Constants/

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